Submitted to Proceedings of 1980 Applied Superconductivity Conf.
Santa Fe, NM

TM-997
1620
September 1980

TRANSMISSION LINE PROPERTIES OF LONG STRINGS OF SUPERCONDUCTING MAGNETS

Robert E. Shafer
Fermi National Accelerator Laboratory*
Batavia, Illinois 60510

1. Introduction

The purpose of this paper is to discuss the electrical characteristics of a long string of superconducting magnets, such as in a superconducting storage ring or accelerator. As the magnets have a shunt capacitance to ground as well as a series inductance, travelling waves can propagate along the string, as in a transmission line. As the string is of finite length, standing waves can also exist. In accelerator quality superconducting magnets, considerable effort has been devoted to minimizing AC losses, the net result being that the magnet string has a high Q precisely at the frequencies which are important for the standing and travelling waves. The magnitude of these effects are estimated, and the solution to be used at Fermilab will be discussed.

2. General Considerations

We assume that the AC power losses in a superconducting magnet can be represented by an electrical equivalent circuit[1]. Specifically, the series impedance of a single magnet may be expressed as

$$Z(\omega) = R(\omega)+j\omega L(\omega) \tag{1}$$

and the shunt admittance (due to capacitance to ground) as

$$Y(\omega) = j\omega C. \tag{2}$$

The solution of the wave equations leads to the defining of a characteristic impedance:

$$Z_o(\omega) = |Z(\omega)/Y(\omega)|^{\frac{1}{2}} \approx \sqrt{L/C} \text{ ohms} \tag{3}$$

and a propagation constant:

$$\gamma(\omega) = \alpha+j\beta = |Z(\omega)\cdot Y(\omega)|^{\frac{1}{2}} \text{ per magnet.} \tag{4}$$

Both the impedance and the propagation constant are complex, frequency dependent quantities. In a travelling wave in the +x direction, the solution is of the general form:

$$V(x,\omega,t) = Z_o I(x,\omega,t) = Z_o I_o e^{-\alpha x} e^{j(\omega t-\beta x)}. \tag{5}$$

Hence the wave has a propagation velocity

$$v(\omega) = \omega/\beta \simeq \sqrt{1/LC} \text{ magnets/sec} \tag{6}$$

and an attenuation length:

$$\lambda = 1/\alpha \approx \frac{2Z_o}{R(\omega)} \text{ magnets to } 1/e. \tag{7}$$

If the magnet string is N magnets long, then standing waves may exist at frequencies ($f = \omega/2\pi$):

$$f_n = \frac{nv(\omega_n)}{N} \tag{8}$$

where n is any odd or even half integer. The damping of standing waves may be measured by the quality factor Q, the number of oscillation cycles to 1/e in power:

$$Q_n = \frac{\omega_n L(\omega_n)}{R(\omega_n)}. \tag{9}$$

3. Application to Fermilab Magnets

At low frequencies ($\lesssim$ 100 Hz), the Fermilab Energy Doubler dipole magnets have an inductance of about 45 mH, a shunt capacitance to ground of about 50 nF, and an effective series resistance (due to eddy currents) of about $R(\omega) = 2.0\times10^{-5}\ \omega^2$ ohms[1]. This leads to the following approximate characteristics:

$$Z_o = 920 \text{ ohms}, \measuredangle = -7^o \quad (120 \text{ Hz}) \tag{10}$$

$$v = 2.0\times10^4 \text{ magnets/sec} \quad (120 \text{ Hz}) \tag{11}$$

$$\lambda = 220 \text{ magnets} \quad (120 \text{ Hz}) \tag{12}$$

$$f_{\frac{1}{2}} = 13 \text{ Hz} \tag{13}$$

$$Q_{\frac{1}{2}} = 30. \tag{14}$$

Note that λ scales roughly as ω^{-2}, and Q as ω^{-1}. We need to introduce a damping mechanism which

(a) does not place any heat load on the cryogenic system,
(b) does not affect dipoles and quadrupoles differently (tune shift)
(c) does not introduce any DC power loss
(d) is passive.

4. Detailed Analysis of Damping Resistor

The solution to be adopted at Fermilab is to include a damping resistor paralleling the coil bus of 4 dipole magnets and 1 quadrupole magnet (one half-cell). The total inductance of this loop is about 186 mH. The connection is made at each voltage monitor tap, which penetrates the cryogenic system at every quadrupole. The main purpose of these taps is to bypass the magnet current in the event of a magnet quench.

Figure 1 details the model of a single dipole magnet, and Figure 2 the damping resistor connection. The quadrupoles are a small effect on the transmission line characteristics, and therefore not included in the model. At every half-cell (quadrupole) the coil and return bus connections are exchanged to balance the inductance of the two lines. The complete string consists of 774 dipoles in series, with the coil and return buses shorted together at the ends. The magnet model used in the analysis is a 6 rather than a 4 terminal device, as although the DC exitation circuit is the coil and return bus, the transmission line is between the coil and ground. Six power supplies are equally spaced in the 774 magnet string. They are in series with the magnet coils and have a low output impedance at the frequencies of interest, hence having little effect on the propagation characteristics of the transmission line mode. The small mutual inductance between the magnet coil and return bus, as well as the effect of the damping resistor loop linking the iron magnet yoke, were considered but not included in the model. The model was compared to impedance measurements on a 16 magnet string, and the agreement was quite good in the range 10 Hz to 3000 Hz.

Analysis of the equivalent circuit was carried out with the aid of a computer program to do frequency

*Operated by Universities Research Association, Inc., under contract with the U.S. Department of Energy.

and transient analysis of large network arrays. The transient analysis included examining the generation and damping of standing waves during the ramping of the magnet string, and the frequency analysis included consideration of ripple current generation at 120, 180, and 720 Hz by power supply ripple voltages, and its attenuation. It should be noted that the damping resistor, as well as the eddy currents inside the magnet, shunt the ripple currents around the central field inductance, hence further reducing the effect of ripple.

The analysis lead to a choice of 100 ohms for the damping resistor. This value of the damping resistor minimizes the attenuation length at 120 Hz, which is expected to be the most serious ripple component. Based on this value, the characteristics of the transmission line are approximately:

$$Z_o = 680 \text{ ohms}, \angle = -28^\circ \quad (120 \text{ Hz}) \tag{15}$$

$$v = 31{,}000 \text{ magnets/sec} \quad (120 \text{ Hz}) \tag{16}$$

$$\lambda = 75 \text{ magnets} \quad (120 \text{ Hz}) \tag{17}$$

$$f_1 = 29 \text{ Hz} \tag{18}$$

$$Q_1 = 2.5 \tag{19}$$

The six terminal nature of the dipole model and the circular (i.e. "racetrack") arrangement of the coil and return bus circuit allows only integral n (i.e. full wave) resonances.

Due to the frequency dependence of $Z(\omega)$ with the damping resistor included, the above parameters are quite frequency dependent, and the transmission line is quite dispersive. At 300 amps/sec (design ramp rate), the current flowing in the damping resistor is about 0.6 amps, leading to an instantaneous power dissipation of 36 watts. As the resistor is external to the magnets, there is no additional load on the cryogenic system. As the current is shunted around both the dipoles and quadrupoles, there is no tune shift.

In terms of the magnetic field, the transfer ratio for the dipole magnets is about 10 Gauss/amp (DC), 9.0 Gauss/amp (120 Hz), and 6.0 Gauss/amp (720 Hz). With the damping resistor installed, the values are 10, 4.2, and 0.9 respectively. The worst case ripple field expected is about 1.3 milliGauss peak during the ramp, and 0.13 milliGauss during flattop (storage) operation.

Reference

1. R.E. Shafer, Fermilab TM-991, 9/22/80

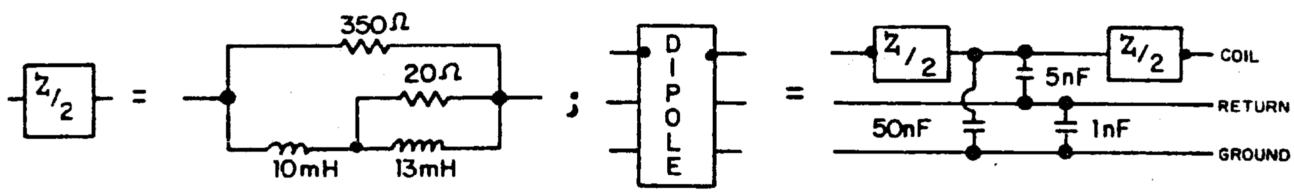

Figure 1. Equivalent Circuit for Fermilab Energy Doubler dipole magnet

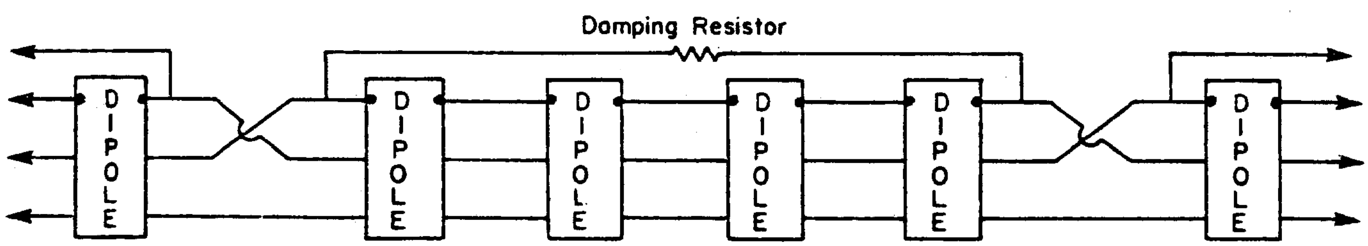

Figure 2. Equivalent circuit for damping resistor connection across 4 dipole magnets